NASA TECHNICAL MEMORANDUM

NASA TM X-73,173

NASA TM X-73,173

A PROGRAM FOR CALCULATING TURBOFAN-DRIVEN LIFT-FAN PROPULSION SYSTEM PERFORMANCE

Michael E. Tauber, Allen E. Fuhs, and John A. Paterson

Ames Research Center
Moffett Field, Calif. 94035

(NASA-TM-X-73173) A PROGRAM FOR CALCULATING TURBOFAN-DRIVEN LIFT-FAN PROPULSION SYSTEM PERFORMANCE (NASA) 26 p HC A03/MF A01 CSCL 21E N77-13061 Unclas G3/07 56991

October 1976

1. Report No. NASA TM X-73,173	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle A PROGRAM FOR CALCULATING TURBOFAN-DRIVEN LIFT-FAN PROPULSION SYSTEM PERFORMANCE		5. Report Date 6. Performing Organization Code
7. Author(s) Michael E. Tauber, Allen E. Fuhs,* and John A. Paterson		8. Performing Organization Report No. A-6776
9. Performing Organization Name and Address Ames Research Center Moffett Field, Calif. 94035		10. Work Unit No. 791-40-04 11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D. C. 20546		13. Type of Report and Period Covered Technical Memorandum 14. Sponsoring Agency Code

15. Supplementary Notes

*Chairman, Department of Mech. Engineering, Naval Postgraduate School, Monterey, California.

16. Abstract

A computer program for calculating the performance of a turbofan-powered lift-fan propulsion system for vertical takeoff and landing (VTOL) aircraft has been written. The program provides quick approximate propulsion system performance information and can be used by persons unfamiliar with the thermodynamics of engine-cycle analysis. The program formulation consists of taking bleed air from a turbofan engine, heating the bleed air in an interburner, and passing it through a tip turbine to drive a lift fan. Two options are available: bleed air from the engine exhaust, or bleed air that has passed through the engine fan only.

17. Key Words (Suggested by Author(s)) Lift-fan propulsion		18. Distribution Statement Unlimited STAR Category - 07	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 26	22. Price* $3.75

*For sale by the National Technical Information Service, Springfield, Virginia 22161

TABLE OF CONTENTS

ii

A PROGRAM FOR CALCULATING TURBOFAN-DRIVEN LIFT-FAN PROPULSION SYSTEM PERFORMANCE

Michael E. Tauber, Allen E. Fuhs,* and John A. Paterson

Ames Research Center

SUMMARY

A computer program for calculating the performance of a turbofan-powered lift-fan propulsion system for vertical takeoff and landing (VTOL) aircraft has been written. The program provides quick approximate propulsion system performance information and can be used by persons unfamiliar with the thermodynamics of engine-cycle analysis.

Since VTOL aircraft propulsion systems are generally sized by takeoff thrust requirements, the program is limited to horizontal and vertical aircraft velocities that are small in comparison with the propulsion exhaust velocities. The program formulation consists of taking bleed air from a turbofan engine, heating the bleed air in an interburner, and passing it through a tip turbine to drive a lift fan. Two options are available: bleed air from the engine exhaust, or bleed air that has passed through the engine fan only.

INTRODUCTION

Option 1: Exhaust-Bleed Drive

The first program option (OPTION = 1) is formulated for air that is bled from the turbofan engine exhaust after the low-pressure fan air and hot core air have been mixed. Although an afterburner can be present, the afterburner is not used during takeoff (to avoid ducting of very high temperature gases). The mass fraction of the exhaust that is bled is one of the program inputs. The bleed air passes through an interburner, to increase its temperature, and then goes through a tip turbine to drive the lift fan. The vertical thrust comes from the lift-fan exhaust, the tip-turbine exhaust, and the engine exhaust by canting the nozzle downward.

Option 2: Fan-Bleed Drive

An alternative to the method of option 1 is to use cool, lower pressure air that has passed through the engine fan only to drive the lift fan (OPTION = 2). It is assumed that all the bypass air that has been compressed

*Chairman, Department of Mech. Engineering, Naval Postgraduate School, Monterey, California.

by the fan is bled, ducted through an interburner to be heated, and continues from there to the tip turbine, thereby driving the lift fan. The vertical thrust, again, comes from the lift-fan and tip-turbine exhaust and the engine central core exhaust, which can be directed downward.

Performance Criteria

The following system performance criteria are calculated: the thrusts of the lift fan, the tip turbine and the engine; the specific thrust and the specific fuel consumption of the entire propulsion system. In addition, the exhaust velocities from the lift fan, tip turbine and engine are given. If required, the corresponding mass flows are readily available by dividing each component thrust by its exhaust velocity.

ASSUMPTIONS

The major assumptions in the analysis relate to the state of the gas, the heating value of the fuel, the operating conditions, and the system component efficiencies. The gas flowing through the power plant is assumed to be calorically and thermally perfect. A calorically perfect gas has constant values of specific heat, both at constant pressure and constant volume; a thermally perfect gas does not dissociate. The heating value of the fuel is assumed to be 18500 BTU/lb; however, the mass flow of fuel is neglected in comparison with the airflow through the engine. Only operation at sea level is considered and at horizontal and vertical aircraft velocities that are small in comparison with the exhaust velocities of the jet engine and lift fan, respectively. A Mach number of 0.4 is used at the main engine face and lift-fan face.

DESCRIPTION OF INPUTS

The inputs begin with a choice of systems: For the engine exhaust-bleed-driven lift fan, the input is OPTION = 1; while for the engine fan-air-bleed-driven lift fan, the input is OPTION = 2. The remaining inputs consist of eight engine or component efficiencies, the limiting turbine inlet temperatures of the engine and tip turbine, the compressor face areas of the engine and of the lift fan, the engine bypass ratio, six total pressure ratios, and the exhaust ratio, defined below. A NAMELIST input format (called DATA) is used, and default values for all inputs are provided. A listing of the inputs follows.

Fortran Name	Variable Description	Units	Default
OPTION = 1	Exhaust-bleed drive	None	1
OPTION = 2	Fan-bleed drive	None	1
ETAFF	Engine-fan efficiency	None	0.85
ETAF	Lift-fan efficiency	None	0.85
ETAC	Compressor efficiency	None	0.85
ETAHT	High-temperature turbine efficiency	None	0.9
ETALT	Low-temperature turbine efficiency	None	0.9
ETAT	Tip-turbine efficiency	None	0.85
ETABB	Engine burner efficiency	None	1.0
ETAB	Interburner efficiency	None	1.0
THTMAX	Maximum engine turbine temperature	°R	3000.
TTMAX	Maximum tip-turbine temperature	°R	2360.
AFF	Engine compressor face area	ft^2	19.63
AF	Lift-fan compressor face area	ft^2	28.27
B	Engine bypass ratio	None	1.0
PIFF	Engine fan compression ratio	None	1.7
PIF	Lift-fan compression ratio	None	1.2
PIC	Engine compressor compression ratio	None	14.7
PIBB	Engine burner pressure loss ratio	None	0.95
PIB	Interburner pressure loss ratio	None	0.90
PIMIX	Engine plenum mixing pressure loss ratio	None	0.90
E	Exhaust ratio: fraction of engine mass flow being exhausted (i.e., not used to drive lift fan)	None	0.5

DESCRIPTION OF OUTPUTS

For each option, there are 10 output quantities to represent thrusts, exhaust velocities, and specific thrust and fuel consumption of the system.

Fortran Name	Variable Description	Units
TF	Lift-fan thrust	lb
TT	Tip-turbine thrust	lb
TE	Thrust of engine exhaust	lb
VF	Lift-fan exhaust velocity	ft/sec
VT	Tip-turbine exhaust velocity	ft/sec
VE	Engine exhaust velocity	ft/sec
I	Specific thrust: total system thrust divided by total system air flow	sec
SFC	Specific fuel consumption: total system fuel flow divided by total system thrust	$\frac{\text{lb fuel}}{\text{lb thrust/hr}}$
R	Ratio of lift-fan plus tip-turbine thrust to engine exhaust thrust	None

Fortran Name	Variable Description	Units
TFF	Total thrust of engine fan (Option 2 only)	lb
M6	Mach number in mixing region for engine fan and core flow (Option 1 only)	None

```
C       LIFTFAN PERFORMANCE PROGRAM
C
C
C
C
C
C
C     ENGINE EXHAUST DRIVEN LIFT FAN (OPTION 1)
C     AND FAN EXHAUST DRIVEN LIFT FAN (OPTION 2)
      REAL M5,M5P,M6,M6PP,M7,MBBMB,I,K,L,MDOT
      G1(X)=1.+F7*X**2
      G2(X)=G1(X)**F8
      G3(X)=.3673*B*G2(X)
      G4(X)=.3993*G2(X)
      G5(X)=F10*G1(X)-1.
      G6(X)=SQRT(G5(X))
      G7(X)=G3(X)/F12
      G8(X)=1.-G4(X)*F16/(F13*G6(X))
      F(X)=G7(X)/G8(X)-X
      NAMELIST /DATA/ OPTION,ETAFF,ETAF,ETAC,ETAHT,ETALT,ETAT,ETABB,
     1 ETAB,THTMAX,TTMAX,AFF,AF,B,PIFF,PIF,PIC,PIBB,PIB,PIMIX,E
      GAM=1.4
      WRITE (6,600)
600   FORMAT(1H1)
C-----SET DEFAULT VALUES FOR INPUT QUANTITIES.
10    OPTION=1
      ETAFF=.85
      ETAF=.85
      ETAC=.85
      ETAHT=.9
      ETALT=.9
      ETAT=.85
      ETABB=1.0
      ETAB=1.0
      THTMAX=3000.
      TTMAX=2360.
      AFF=19.63
      AF=28.27
      B=1.0
      PIFF=1.7
      PIF=1.2
      PIC=14.7
      PIBB=.95
      PIB=.9
      PIMIX=.9
      E=.5
C-----READ INPUT QUANTITIES.
      READ (5,DATA)
      IF (OPTION.NE.1) GO TO 100
```

```
C-------------------------------------------------------
C       OPTION 1 (ENGINE EXHAUST DRIVEN LIFT FAN).
C-------------------------------------------------------
        WRITE (6,601) ETAFF,ETAF,ETAC,ETAHT,ETALT,ETAT,ETABB,ETAB,THTMAX,
       1 TTMAX,AFF,AF,B,PIFF,PIF,PIC,PIBB,PIB,PIMIX,E
 601    FORMAT(////35X,30HENGINE EXHAUST DRIVEN LIFT FAN//
       1 9H    ETAFF=F6.3,9H     ETAF=F6.3,9H     ETAC=F6.3,9H    ETAHT=F6.3,
       2 9H    ETALT=F6.3,9H     ETAT=F6.3,9H    ETABB=F6.3,9H     ETAB=F6.3/
       3 9H   THTMAX=F6.1,9H    TTMAX=F6.1,9H      AFF=F6.3,9H       AF=F6.3,
       4 9H        B=F6.3,9H     PIFF=F6.3,9H      PIF=F6.3,9H      PIC=F6.3/
       5 9H     PIBB=F6.3,9H      PIB=F6.3,9H    PIMIX=F6.3,9H        E=F6.3)
        F1=(GAM-1.)/GAM
        TAUF=1.+(PIF**F1-1.)/ETAF
        F2=TAUF-1.
        IF (F2.GT.0.) GO TO 15
        WRITE (6,602) F2
 602    FORMAT(/4H F2=,E13.5,3X,19HF2 SET EQUAL TO 0.5/)
        F2=0.5
 15     TF=2434.*AF*SQRT(F2)
        F3=(PIFF**F1-1.)/ETAFF
        F4=(PIC**F1-1.)/ETAC
        F5=1.+B
        F6=F5/ETALT
        PILTG=1.-F6*F3/(THTMAX/519.-(F3+1.)*F4)
        PIHTG=1.-519.*(F3+1.)*F4/(THTMAX*ETAHT)
        F7=(GAM-1.)/2.
        F8=(GAM+1.)/(2.*(GAM-1.))
        F9=.5*F1
        F10=PILTG*PIHTG*(PIBB*PIC)**F1
        F11=SQRT(F10)
        F12=F5*PIFF**F1
        F13=F5*PIFF*F11
        F14=1.-ETALT*(1.-PILTG)
        F15=1.-ETAHT*(1.-PIHTG)
        IF (F14.GT.0..AND.F15.GT.0.) GO TO 20
        WRITE (6,603)
 603    FORMAT(/60H M5P CANNOT BE FOUND. CYCLE INOPERATIVE. GOING TO NEXT
       1CASE.)
        GO TO 10
 20     F16=SQRT(F14*F15)
C-----SEARCH FOR ROOT TO TRANSCENDENTAL EQUATION TO GET VALUE FOR M5P.
        X1=0.05
        X2=1.0
        IF (F(X1)*F(X2).LT.0.) GO TO 25
        WRITE (6,603)
        GO TO 10
 25     X=.5*(X1+X2)
        DO 35 J=1,8
```

```
      IF (G5(X1).GT.0..AND.G5(X2).GT.0..AND.G5(X).GT.0.) GO TO 30
      WRITE (6,603)
      GO TO 10
30    Y1=F(X1)
      Y2=F(X2)
      Y=F(X)
      IF (Y1*Y.GT.0.) X1=X
      IF (Y1*Y.LT.0.) X2=X
      IF (Y1*Y.EQ.0.) GO TO 40
      X=.5*(X1+X2)
35    CONTINUE
C-----END OF SEARCH FOR ROOT.
40    M5P=X
      F17=1./F1
      P5P=2116.*PIFF/G1(M5P)**F17
      F18=5.*(F10*(PIFF*2116./P5P)**F1-1.)
      IF (F18.GT.0.) GO TO 45
      WRITE (6,605) F18
605   FORMAT(/5H F18=,E13.5,3X,20HF18 SET EQUAL TO 0.5/)
      F18=0.5
45    M5=SQRT(F18)
      T5=THTMAX*F14*F15/G1(M5)
      V5=49.*M5*SQRT(T5)
      T5P=519.*PIFF**F1/G1(M5P)
      V5P=49.*M5P*SQRT(T5P)
      G=AFF*P5P+.9748*AFF*(V5+V5P*B)/F5
      TT5=T5*G1(M5)
      TT1=519.*(1.+(PIFF**F1-1.)/ETAFF)
      TT6=(TT5+B*TT1)/F5
      K=47.77*AFF*SQRT(TT6)/G
      IF (K.LT.SQRT(.5*GAM)) GO TO 50
      WRITE (6,606) K
606   FORMAT(/ 3H K=E13.5,3X,60HNO SOLUTION TO MIXING MACH NO. EQUATION.
     1 GOING TO NEXT CASE.)
      GO TO 10
50    F19=2.*K**2-GAM
      C1=2.*F19/(GAM*(F19+1.))
      C2=2.*K**2/(GAM**2*(F19+1.))
      F20=C1**2-4.*C2
      IF (F20.GT.0.) GO TO 55
      WRITE (6,607) F20
607   FORMAT(/5H F20=,E13.5,3X,20HF20 SET EQUAL TO 0.5/)
      F20=0.5
55    F21=(SQRT(F20)-C1)/2.
      IF (F21.GT.0.) GO TO 60
      WRITE (6,608) F21
608   FORMAT(/5H F21=,E13.5,3X,20HF21 SET EQUAL TO 0.5/)
      F21=0.5
```

```
60      M6=SQRT(F21)
        T6=TT6/(1.+F7*M6)
        P6=G/(AFF*(1.+GAM*M6**2))
        PT6=P6*G1(M6)**F17
        F22=5.*((P6/2116.)**F1*G1(M6)-1.)
        IF (F22.GT.0.) GO TO 65
        WRITE (6,609) F22
609     FORMAT(/5H F22=,E13.5,3X,20HF22 SET EQUAL TO 0.5/)
        F22=0.5
65      M6PP=SQRT(F22)
        TE=47.77*AFF*E*M6PP*SQRT(TT6/G1(M6PP))
        F23=1.-519.*AF*(1.-TAUF)/(TTMAX*ETAT*AFF*(1.-E))
        PT72G=(PIB*PIMIX*PIFF)**F1*F23
        F24=5.*(PT72G-1.)
        IF (F24.GT.0.) GO TO 70
        WRITE (6,610) F24
610     FORMAT(/5H F24=,E13.5,3X,20HF24 SET EQUAL TO 0.5/)
        F24=0.5
70      M7=SQRT(F24)
        F25=1.-519.*AF*(1.-TAUF)/(TTMAX*AFF*(1.-E))
        T7=TTMAX*F25/G1(M7)
        IF (T7.GT.0.) GO TO 75
        WRITE (6,611) T7
611     FORMAT(/4H T7=,E13.5,3X,19HT7 SET EQUAL TO 0.5/)
        T7=0.5
75      TT=47.77*AFF*M7*(1.-E)*SQRT(T7)
        R=(TF+TT)/TE
        F26=TF+TT+TE
        I=F26/(31.4*(AFF+AF))
        TT2=519.*(1.+F4)*(1.+F3)
        F27=TT2*(THTMAX/TT2-1.)/(ETABB*F5)
        F28=TT6*(1.-E)*(TTMAX/TT6-1.)/ETAB
        MBBMB=1.265E-5*AFF*(F27+F28)
        SFC=1.159E5*MBBMB/F26
        IF (F2.GT.0.) GO TO 80
        WRITE (6,602) F2
80      VF=2497.*SQRT(F2)
        VE=49.*M6PP*SQRT(TT6/G1(M6PP))
        IF (T7.GT.0.) GO TO 85
        WRITE (6,611) T7
85      VT=49.*M7*SQRT(T7)
        WRITE (6,612) TF,TT,TE,VF,VT,VE,I,SFC,R,M6
612     FORMAT(/ 5X,3HTF=1PE10.3,5H (LB),9X,3HTT=E10.3,5H (LB),
       1 5X,3HTE=E10.3,5H (LB),9X,3HVF=E10.3,9H (FT/SEC)/
       2 5X,3HVT=E10.3,9H (FT/SEC),5X,3HVE=E10.3,9H (FT/SEC),
       3 6X,2HI=E10.3,6H (SEC),7X,4HSFC=E10.3,6H (/HR)/
       4 6X,2HR=E10.3,14X,3HM6=E10.3)
        GO TO 10
```

```
C-----------------------------------------------------------------
C     OPTION 2 (FAN EXHAUST DRIVEN LIFT FAN).
C-----------------------------------------------------------------
100   WRITE (6,650) ETAFF,ETAF,ETAC,ETAHT,ETALT,ETAT,ETABB,ETAB,THTMAX,
     1 TTMAX,AFF,AF,B,PIFF,PIF,PIC,PIBB,PIB
650   FORMAT(////35X,27HFAN EXHAUST DRIVEN LIFT FAN//
     1 9H   ETAFF=F6.3,9H    ETAF=F6.3,9H    ETAC=F6.3,9H   ETAHT=F6.3,
     2 9H   ETALT=F6.3,9H    ETAT=F6.3,9H   ETABB=F6.3,9H    ETAB=F6.3/
     3 9H  THTMAX=F6.1,9H   TTMAX=F6.1,9H     AFF=F6.3,9H      AF=F6.3,
     4 9H       B=F6.3,9H    PIFF=F6.3,9H     PIF=F6.3,9H     PIC=F6.3/
     5 9H    PIBB=F6.3,9H     PIB=F6.3)
      C1=(GAM-1.)/GAM
      C2=1.+B
      L=AF*C2/(AFF*B)
      MDOTT=.975*AF/L
      C3=(PIF**C1-1.)/ETAF
      C4=SQRT(C3)
      C5=(PIFF**C1-1.)/ETAFF
      C6=SQRT(C5)
      C7=(PIC**C1-1.)/ETAC
      C8=519.*L*C3/TTMAX
      C9=(PIB*PIFF)**C1*(1.-C8/ETAT)
      C10=C2/ETALT
      C11=1.+C5
      C12=1.-519.*C11*C7/(THTMAX*ETAHT)
      C13=1.-C10*C5/(THTMAX/519.-C11*C7)
      C14=C12*C13
      C15=(PIBB*PIC*PIFF)**C1
      C16=1.-519.*C11*C7/THTMAX
      C17=1.-C2*C5/(THTMAX/519.-C11*C7)
      C18=(1.-C8)*(1.-1./C9)
      C19=(C14*C15-1.)*C16*C17/(C14*C15)
      IF (C18.GT.0..AND.C19.GT.0.) GO TO 105
      WRITE (6,651) C18,C19
651   FORMAT(/ 3X,4HC16=E11.4,3X,4HC19=E11.4,3X,
     1 20H GOING TO NEXT CASE.)
      GO TO 10
105   TF=2432.*AF*C4
      VF=2495.*C4
      VT=5245.*SQRT(C18)
      VE=6003.*SQRT(C19)
      TT=.975*AFF*VT/C2
      TE=.975*AFF*VE/C2
      I=.031*(B*(L*VF+VT)+VE)/(C2+L*B)
      ABB=(6.733E-3/ETABB)*(THTMAX/519.-C11*(1.+C7))
      AB=(6.733E-3/ETAB)*(TTMAX/519.-C11)
      SFC=1.159E5*(ABB+B*AB)/(B*(L*VF+VT)+VE)
      R=B*(L*VF+VT)/VE
```

ORIGINAL PAGE IS
OF POOR QUALITY

```
      TFF=2432.*AFF*C6*B/C2
      WRITE (6,652) TF,TT,TE,VF,VT,VE,I,SFC,R,TFF
652   FORMAT(/ 5X,3HTF=1PE10.3,5H (LB),9X,3HTT=E10.3,5H (LB),
     1 9X,3HTE=E10.3,5H (LB),9X,3HVF=E10.3,9H (FT/SEC)/
     2 5X,3HVT=E10.3,9H (FT/SEC),5X,3HVE=E10.3,9H (FT/SEC),
     3 6X,2HI=E10.3,6H (SEC),7X,4HSFC=E10.3,6H (/HR)/
     4 6X,2HR=E10.3,13X,4HTFF=E10.3,5H (LB))
      GO TO 10
      END
```

ENGINE EXHAUST DRIVEN LIFT FAN

```
ETAFF= 0.850     ETAF= 0.850     ETAC= 0.850    ETAHI= 0.900   ETALT= 0.900    ETAT= 0.850    [illegible]= 1.[illegible]    [illegible]= 1.[illegible]
THTMAX=3000.0    TTMAX=2360.0    AFF=19.630     AF=28.270      R= 1.000        PIFF= 1.700    PIF= 1.200     [illegible]=14.7[illegible]
PIBB= 0.950      PIB= 0.900      PIMIX= 0.900   L= 0.500

TF= 1.726E 04 (LB)             TT= 1.860E 04 (LB)           TE= 1.668E 04 (LB)        VE= 6.253E 02 (FT/SEC)
VT= 1.944E 03 (FT/SEC)         VE= 1.743E 03 (FT/SEC)       T= 3.493E 01 (SEC)        SFC= 7.151E-01 (/HR)
R= 2.149E 00                   M6= 2.824E-01
```

FAN EXHAUST DRIVEN LIFT FAN

```
ETAFF= 0.850     ETAF= 0.850     ETAC= 0.850    ETAHI= 0.900   ETALT= 0.900    ETAT= 0.850    [illegible]= 1.[illegible]    [illegible]= 1.[illegible]
THTMAX=3000.0    TTMAX=2360.0    AFF=19.630     AF=28.270      R= 1.000        PIFF= 1.700    PIF= 1.200     [illegible]=14.7[illegible]
PIBB= 0.950      PIB= 0.900

TF= 1.724E 04 (LB)             TT= 1.309E 04 (LB)           TF= 2.726E 04 (LB)        VF= 6.256E 02 (FT/SEC)
VT= 1.369E 03 (FT/SEC)         VF= 2.849E 03 (FT/SEC)       T= 3.823E 01 (SEC)        SFC= 8.195E-01 (/HR)
R= 1.113E 00                   TFF= 1.048E 04 (LB)
```

APPENDIX

DERIVATION OF EQUATIONS

SYMBOLS

A fan area, ft^2

a speed of sound, ft/sec

B bypass ratio

C_1 see equation (7)

C_2 see equation (7)

c_p specific heat at constant pressure, $\frac{BTU}{lb°R}$

c_v specific heat at constant volume, $\frac{BTU}{lb°R}$

E exhaust ratio

G momentum flux, $\frac{lb\text{-}sec}{sec}$

g acceleration of gravity, 32.2 ft/sec^2

H heating value of fuel, 18500 BTU/lb

I specific thrust of entire propulsion system, lb thrust/lb/sec of air flow

K see equation (7)

M Mach number

$\dot{m}$ mass flow, slugs/sec

p pressure, lb/ft^2

R gas constant for air, 1716 $ft^2/sec^2°R$; also ratio of lift fan plus tip turbine thrust to engine thrust

SFC specific fuel consumption of entire propulsion system, lb/lb/hr

T temperature, °R, or thrust, lb

V velocity, ft/sec

γ specific heat ratio

η efficiency

π ratio of total pressures

τ ratio of total temperatures

ρ density of air, slugs/ft^3

Subscripts*

t total conditions

∞ ambient conditions

O condition at fan face

B interburner

BB engine burner

C engine compressor

E engine exhaust

F lift fan

FF engine fan

HT high-temperature turbine in engine

LT lower temperature turbine in engine

MIX region of mixing between fan and core air in engine

T tip turbine

ASSUMPTIONS

The following basic assumptions are made in the derivations:

(a) Calorically perfect gas

$$c_p = \text{constant}$$

$$\gamma = \frac{c_p}{c_v} = \text{constant}$$

*Note: For number subscripts, refer to figures.

(b) Thermally perfect gas

$$p = \rho RT$$

(c) Fuel-to-air ratio is small, typically about 0.02, and fuel mass can be neglected in comparison with air mass.

The static pressures in the exhaust of the engine, the lift fan, and the tip turbine are all assumed to be atmospheric. Thus, to calculate the thrusts, we must determine the exhaust velocities.

EXHAUST-BLEED DRIVE SYSTEM FORMULATION (OPTION 1)

For the engine and the tip turbine (fig. 1), the exhaust velocities depend on the velocity in the region where the air that has passed through the engine fan is mixed with the hot, higher pressure engine core air.

Conditions in the mixing region are calculated by using the continuity, momentum, and energy equations, with the assumption that there is constant area mixing. The continuity equation is

$$\rho_5 V_5 A_5 + \rho_{5'} V_{5'} A_{5'} = \rho_6 V_6 A_{FF} = \dot{m}_C + \dot{m}_{\Delta FF} \tag{1}$$

where $\dot{m}_C$ is the mass flow through the engine core and $\dot{m}_{\Delta FF}$ is the mass flow through the engine fan only. The momentum equation is

$$A_{FF} p_5 + \rho_5 V_5^2 A_5 + \rho_{5'} V_{5'}^2 A_{5'} = A_{FF} p_6 + \rho_6 V_6^2 A_{FF} = G \tag{2}$$

where G is the total momentum flux and it is assumed that

$$p_5 = p_{5'}$$

The energy equation is

$$\dot{m}_C c_p T_{t_5} + \dot{m}_{\Delta FF} c_p T_{t_{5'}} = (\dot{m}_C + \dot{m}_{\Delta FF}) c_p T_{t_6} \tag{3}$$

where it is assumed that

$$T_{t_{5'}} = T_{t_1}$$

The static pressure in the mixing region can be written in terms of the momentum flux as

$$p_6 = \frac{G}{A_{FF}(1 + \gamma M_6^2)} \tag{4}$$

and the mass flow per unit area as

$$\rho_6 V_6 = \frac{\gamma M_6 \sqrt{1 + \frac{\gamma - 1}{2} M_6^2}\; G}{A_{FF}\sqrt{\gamma R T_{t_6}}\,(1 + \gamma M_6^2)} \tag{5}$$

Substitution of the continuity equation (eq. (1)) into equation (5) leads to a polynomial in the mixing Mach number, M_6

$$\frac{2\gamma K^2 - \gamma(\gamma - 1)}{2} M_6^4 + (2K^2 - \gamma)M_6^2 + \frac{K^2}{\gamma} = 0 \tag{6}$$

which has the solution

$$M_6 = \left[\frac{-C_1 + \sqrt{C_1^2 - 4C_2}}{2}\right]^{1/2} \tag{7}$$

where

$$K = \frac{(\dot{m}_C + \dot{m}_{\Delta FF})\sqrt{\gamma R T_{t_6}}}{G}$$

$$C_1 = \frac{2(2K^2 - \gamma)}{\gamma(2K^2 - \gamma + 1)}$$

$$C_2 = \frac{2K^2}{\gamma^2(2K^2 - \gamma + 1)}$$

Equation (7) has physically meaningful solutions only if

$$K < \sqrt{\frac{\gamma}{2}}\ .$$

Before equation (7) can be solved, T_{t_6}, $p_{5'}$, $V_{5'}$, and V_5 must be found. Since T_{t_6} can be written

$$T_{t_6} = \frac{\dot{m}_C T_{t_5} + \dot{m}_{\Delta FF} T_{t_1}}{\dot{m}_C + \dot{m}_{\Delta FF}} \tag{8}$$

the solution to equation (7), therefore, depends on conditions at stations 5 and 5'.

The following efficiencies are defined:

For fans and compressors

$$\eta = \frac{\pi^{(\gamma-1)/\gamma} - 1}{\tau - 1}$$

and for turbines

$$\eta = \frac{1 - \tau}{1 - \pi^{(\gamma-1)/\gamma}}$$

where τ is the ratio of total temperatures and π is the ratio of total pressures across the component.

The conditions at station 5 are given by

$$V_5 = M_5\sqrt{\gamma R T_5} \tag{9}$$

$$T_5 = \frac{T_{t_3}^{MAX}\left[1 \quad \text{[illegible]}\left(1 - \pi_{LT}^{(\gamma-1)/\gamma}\right)\right]\left[1 - \eta_{HT}\left(1 - \pi_{HT}^{(\gamma-1)/\gamma}\right)\right]}{1 + \frac{\gamma - 1}{2} M_5{}^2} \tag{10}$$

where $T_{t_3}^{MAX}$ is the maximum turbine inlet temperature, and the turbine pressure ratios are

$$\pi_{LT}^{(\gamma-1)/\gamma} = 1 - \frac{\left(\frac{1 + B}{\eta_{LT}}\right)\left(\frac{\pi_{FF}^{(\gamma-1)/\gamma} - 1}{\eta_{FF}}\right)}{\frac{T_{t_3}^{MAX}}{T_\infty} - \left(1 + \frac{\pi_{FF}^{(\gamma-1)/\gamma} - 1}{\eta_{FF}}\right)\left(\frac{\pi_C^{(\gamma-1)/\gamma} - 1}{\eta_C}\right)} \tag{11}$$

$$\pi_{HT}^{(\gamma-1)/\gamma} = 1 - \frac{T_\infty}{T_{t_3}^{MAX}\ \eta_{HT}}\left(1 + \frac{\pi_{FF}^{(\gamma-1)/\gamma} - 1}{\eta_{FF}}\right)\left(\frac{\pi_C^{(\gamma-1)/\gamma} - 1}{\eta_C}\right) \tag{12}$$

The Mach number is

$$M_5 = \left\{\frac{2}{\gamma - 1}\left[\left(\pi_{LT}\pi_{HT}\pi_{BB}\pi_C\pi_{FF}\ \frac{p_\infty}{p_{5'}}\right)^{(\gamma-1)/\gamma} - 1\right]\right\}^{1/2} \tag{13}$$

However, M_5 is a function of $p_{5'}$ which depends on $M_{5'}$ through

$$p_{5'} = \frac{p_\infty \pi_{FF}}{\left(1 + \frac{\gamma - 1}{2} M_{5'}^2\right)^{\gamma/(\gamma-1)}} \tag{14}$$

and

$$M_{5'} = \frac{0.3673B\left(1 + \frac{\gamma - 1}{2} M_{5'}^2\right)^{(\gamma+1)/2(\gamma-1)}}{(1 + B)\pi_F^{(\gamma-1)/\gamma}\left\{1 - \frac{0.3993\left(1 + \frac{\gamma - 1}{2} M_{5'}^2\right)^{(\gamma+1)/2(\gamma-1)}\left[1 - \eta_{LT}\left(1 - \pi_{LT}^{(\gamma-1)/\gamma}\right)\right]^{1/2}\left[1 - \eta_{LT}\left(1 - \pi_{HT}^{(\gamma-1)/\gamma}\right)\right]^{1/2}}{(1 + B)\pi_F(\pi_{LT}\pi_{HT}\pi_B\pi_C)^{(\gamma-1)/2\gamma}\left[(\pi_{LT}\pi_{HT}\pi_B\pi_C)^{(\gamma-1)/\gamma}\left(1 + \frac{\gamma - 1}{2} M_{5'}^2\right) - 1\right]^{1/2}}\right\}} \tag{15}$$

Equation (15) is a transcendental equation which is solved iteratively. Now $T_{5'}$ can also be found

$$T_{5'} = \frac{T_\infty \pi_{FF}^{(\gamma-1)/\gamma}}{1 + \frac{\gamma - 1}{2} M_{5'}^2} \tag{16}$$

The velocity of the hot core air is

$$V_{5'} = M_{5'}\sqrt{\gamma R T_{5'}} \tag{17}$$

while the total temperatures of the hot core air and cooler fan air are, respectively

$$T_{t_5} = T_5\left(1 + \frac{\gamma - 1}{2} M_5^2\right) \tag{18}$$

and

$$T_{t_1} = T_{t_{5'}} = T_\infty\left(1 + \frac{\pi_{FF}^{(\gamma-1)/\gamma} - 1}{\eta_{FF}}\right) . \tag{19}$$

The total temperature in the mixing region is given by equation (8), and the static temperature is

$$T_6 = \frac{T_{t_6}}{1 + \frac{\gamma - 1}{2} M_6^2} \tag{20}$$

The static pressure in the mixing region is given by equation (4) and the total pressure is

$$p_{t_6} = p_6\left(1 + \frac{\gamma - 1}{2} M_6{}^2\right)^{\gamma/(\gamma-1)} \tag{21}$$

The thrust of the engine exhaust, neglecting turning losses, is

$$T_E = \rho_\circ V_\circ A_{FF} E M_{6''} \left(\frac{\gamma R T_{t_6}}{1 + \frac{\gamma - 1}{2} M_{6''}^2}\right)^{1/2} \tag{22}$$

where the exhaust Mach number is

$$M_{6''} = \left\{\frac{2}{\gamma - 1}\left[\left(\frac{p_6}{p_\infty}\right)^{(\gamma-1)/\gamma}\left(1 + \frac{\gamma - 1}{2} M_6{}^2\right) - 1\right]\right\}^{1/2} \tag{23}$$

The engine exhaust is bled, passes through an interburner to be heated, and then goes through a tip turbine to drive the lift fan. The work balance for the tip turbine and lift fan is

$$\dot{m}_T c_p\left(T_{t_{6'}} - T_{t_7}\right) = \dot{m}_F c_p\left(T_{t_9} - T_{t_8}\right) \tag{24}$$

where

$$T_{t_8} = T_\infty \ .$$

If the tip-turbine and lift-fan exhausts are expanded to ambient pressure

$$p_8 = p_{10} = p_\infty$$

and the thrusts of the lift fan and tip turbine, respectively, are

$$T_F = \dot{m}_F V_9 \tag{25}$$

$$T_T = \dot{m}_T V_7 \ . \tag{26}$$

The exhaust Mach number of the lift fan is

$$M_9 = \frac{2}{\gamma - 1}\,(1 - \tau_F) \tag{27}$$

where

$$\tau_F = 1 + \frac{\pi_F^{(\gamma-1)/\gamma} - 1}{\eta_F}$$

and the lift-fan thrust is

$$T_F = \rho_o V_o A_F \sqrt{\frac{2\gamma}{\gamma - 1} RT_\infty (\tau_F - 1)} \quad . \tag{28}$$

The Mach number of the tip-turbine exhaust is

$$M_7 = \sqrt{\frac{2}{\gamma - 1}\left[\left(\frac{p_{t_7}}{p_\infty}\right)^{(\gamma-1)/\gamma} - 1\right]} \tag{29}$$

where

$$\left(\frac{p_{t_7}}{p_\infty}\right)^{(\gamma-1)/\gamma} = \left(\pi_B \pi_{FF} \pi_{MIX}\right)^{(\gamma-1)/\gamma}\left[1 - \frac{T_\infty A_F (1 - \tau_F)}{T_{t_6'}^{MAX} \eta_T A_{FF} (1 - E)}\right] \tag{30}$$

and $T_{t_6'}^{MAX}$ is the maximum interburner temperature. The thrust of the tip-turbine exhaust is

$$T_T = \rho_o V_o A_F M_7 (1 - E) \sqrt{\gamma R T_7} \tag{31}$$

where

$$T_7 = \frac{T_{t_6'}^{MAX}}{1 + \frac{\gamma - 1}{2} M_7^2}\left[1 - \frac{T_\infty A_F (1 - \tau_F)}{T_{t_6'}^{MAX} A_{FF} (1 - E)}\right] \tag{32}$$

The specific thrust is defined here as the total thrust of the propulsion system divided by the total airflow through the system:

$$I = \frac{T_F + T_T + T_E}{g\rho_o V_o (A_F + A_{FF})} \quad . \tag{33}$$

The thrust ratio is defined as

$$R = \frac{T_F + T_T}{T_E} \quad . \tag{34}$$

The specific fuel consumption of the propulsion system is

$$\mathrm{SFC} = \frac{3600g(\dot{m}_B + \dot{m}_{BB})}{T_F + T_T + T_E} \tag{35}$$

where

$$\dot{m}_{BB} + \dot{m}_B = \frac{\rho_\circ V_\circ A_{FF} c_p}{H}\left[\frac{T_{t_2}}{\eta_{BB}(1 + B)}\left(\frac{T_{t_3}^{MAX}}{T_{t_2}} - 1\right) + \frac{T_{t_6}(1 - E)}{\eta_B}\left(\frac{T_{t_{6'}}^{MAX}}{T_{t_6}} - 1\right)\right] \tag{36}$$

and

$$T_{t_2} = T_\infty\left(1 + \frac{\pi_C^{(\gamma-1)/\gamma} - 1}{\eta_C}\right)\left(1 + \frac{\pi_{FF}^{(\gamma-1)/\gamma} - 1}{\eta_{FF}}\right) \tag{37}$$

FAN-BLEED DRIVE SYSTEM FORMULATION (OPTION 2)

It is assumed that all the bypass air is bled, passed through an interburner to be heated, and goes through the tip turbine to drive the lift fan (see fig. 2).

The thrust of the lift fan is given by equations (25), (27), and (28). The thrust of the engine fan is

$$T_{FF} = \frac{B\rho_\circ V_\circ A_{FF}}{B + 1}\sqrt{\frac{2\gamma}{\gamma - 1} RT_\infty\left(\frac{\pi_{FF}^{(\gamma-1)/\gamma} - 1}{\eta_{FF}}\right)} \tag{38}$$

The thrust of the tip turbine is given by

$$T_T = \dot{m}_T V_7$$

where

$$\dot{m}_T = \frac{\rho_\circ V_\circ A_{FF} B}{B + 1} \tag{39}$$

and

$$V_7 = \left(\frac{2\gamma R}{\gamma - 1} T_{t_{6'}}^{MAX}\right)^{1/2}\left[1 - \frac{LT_\infty}{T_{t_{6'}}^{MAX}}\left(\frac{\pi_F^{(\gamma-1)/\gamma} - 1}{\eta_F}\right)\right]^{1/2} \times$$

$$\left\{1 - \frac{1}{(\pi_B\pi_{FF})^{(\gamma-1)/\gamma}\left[1 - \frac{LT_\infty}{\eta_T T_{t_{6'}}^{MAX}}\left(\frac{\pi_F^{(\gamma-1)/\gamma} - 1}{\eta_F}\right)\right]}\right\}^{1/2} \tag{40}$$

Here, L is lift-fan ratio, defined as

$$L = \frac{\dot{m}_F}{\dot{m}_T} = \frac{A_F(B + 1)}{A_{FF}B} \tag{41}$$

The thrust of the engine core flow is

$$T_E = \dot{m}_C V_5$$

where

$$\dot{m}_C = \frac{\rho_\circ V_\circ A_{FF}}{B + 1} \tag{42}$$

The engine exhaust velocity is

$$V_5 = \left(\frac{2\gamma R}{\gamma - 1} T_{t_3}^{MAX}\right)^{1/2}\left[(\pi_{HT}\pi_{LT})^{(\gamma-1)/\gamma}(\pi_{BB}\pi_C\pi_{FF})^{(\gamma-1)/\gamma} - 1\right]^{1/2} \times$$

$$\left[\frac{\tau_{HT}\tau_{LT}}{(\pi_{HT}\pi_{LT})^{(\gamma-1)/\gamma}(\pi_{BB}\pi_C\pi_{FF})^{(\gamma-1)/\gamma}}\right]^{1/2} \tag{43}$$

where $\pi_{LT}^{(\gamma-1)/\gamma}$ and $\pi_{HT}^{(\gamma-1)/\gamma}$ are given by equations (11) and (12), respectively, and

$$\tau_{HT} = 1 - \frac{T_\infty}{T_{t_3}^{MAX}}\left(1 + \frac{\pi_{FF}^{(\gamma-1)/\gamma} - 1}{\eta_{FF}}\right)\left(\frac{\pi_C^{(\gamma-1)/\gamma} - 1}{\eta_C}\right) \tag{44}$$

$$\tau_{LT} = 1 - \frac{(1 + B)\left(\dfrac{\pi_{FF}^{(\gamma-1)/\gamma} - 1}{\eta_{FF}}\right)}{\dfrac{T_\infty}{T_{t_3}^{MAX}} - \left(1 + \dfrac{\pi_{FF}^{(\gamma-1)/\gamma} - 1}{\eta_{FF}}\right)\left(\dfrac{\pi_C^{(\gamma-1)/\gamma} - 1}{\eta_C}\right)} \tag{45}$$

The expressions for specific thrust, thrust ratio, and SFC are the same as given in equations (33), (34), and (35); however, the mass flow through the interburner is different. The expression which replaces equation (36) is

$$\dot{m}_{BB} + \dot{m}_B = \frac{\rho_0 V_0 A_{FF} c_p T_\infty}{(B + 1)H} \left\{ \frac{1}{\eta_{BB}} \left[\frac{T_{t_3}^{MAX}}{T_\infty} - \left(1 + \frac{\pi_{FF}^{(\gamma-1)/\gamma} - 1}{\eta_{FF}}\right)\left(1 + \frac{\pi_C^{(\gamma-1)/\gamma} - 1}{\eta_C}\right) \right] + \frac{B}{\eta_B} \left[\frac{T_{t_{6'}}^{MAX}}{T_\infty} - \left(1 + \frac{\pi_{FF}^{(\gamma-1)/\gamma} - 1}{\eta_{FF}}\right) \right] \right\} \tag{46}$$

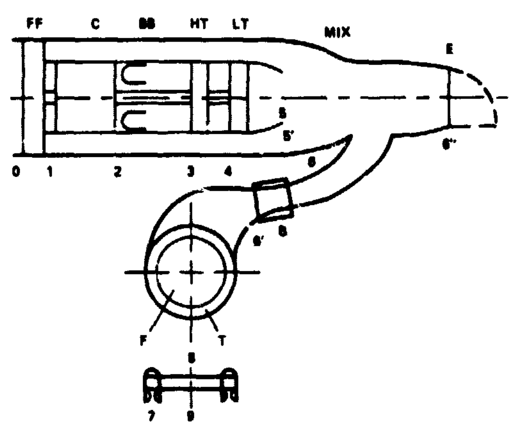

Figure 1.- Option 1: exhaust-bleed drive.

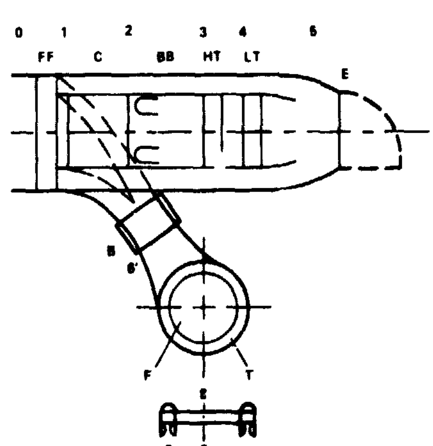

Figure 2.- Option 2: fan-bleed drive.